

## RESEÑAS / BOOK REVIEWS

### BIFURCATIONS OF PLANAR VECTOR FIELDS AND HILBERT'S SIXT PROBLEM

R. Roussarie, Basel-Boston-Berlin, Birkhäuser Verlag, Progress in Mathematics Vol. 164  
USC 1995: 58F14; 35B32

**Key words:** bifurcations, planar vector fields, limit periodic sets

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This text was prepared the support the lectures given in I.M.P.A. during the 20<sup>th</sup> Colóquio Brasileiro de Matemática, in July 1995. The declared subject of this book is the systematic study of bifurcations of regular, elementary and non-elementary limit periodic sets in families of planar vector fields  $(X_\lambda)$  depending on a parameter  $\lambda \in \mathbb{R}^k$ . Furthermore, is presented the Hilbert's sixteenth problem by the study of all possible limit periodic sets in polynomial families and the study of the cyclicity of these sets, on the basis of a more general conjecture about the finite cyclicity of analytic unfoldings of limit periodic sets. In the text is proved that this conjecture would imply a positive answer to the Hilbert's problem:

**"For any  $n \geq 2$  there exists a finite number  $H(n)$  such that any polynomial vector field of degree  $\leq n$  has less than  $H(n)$  limit cycles".**

Chapter one is devoted to general presentation of families of two-dimensional vector fields on surfaces of genus 0, and is given also a first approach to bifurcation theory.

Chapter two is devoted to the notion and structure of limit periodic sets for vector fields. The cyclicity property allows to replace the problem of finding a uniform bound for the number of limit cycles of a given family by a local problem on the number of limit cycles wich bifurcate from each limit periodic set. This definition is given in this chapter complemented with various examples.

The blowing up technique for the study of singularities of vector fields and related procedures are treated in Chapter Three.

Periodic orbits and elliptic singular points which are limits of sequences of limit cycles are called regular limit periodic sets. Bifurcations of such sets are studied in Chapter Four. One can read here some applications to the suty of quadratic vector fields.

The simplest limit periodic sets after the regular ones are the so-called elementary graphics. Bifurcations of elementary graphics are presented in Chapter Five. Very interesting presentations of unfoldings of saddle connections in the finite codimension and in the infinite codimension cases are developed here.

The basis of the desingularization theory and bifurcation on non-elementary limit periodic sets are studied in Chapter Six. Particularly, is presented a method of desingularization for analytic vector fields due to Dumortier, El Morsalini and Rousseau related with the Hilbert's 16<sup>th</sup> Problem for quadratic vector fields.

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# COMPUTATIONAL GEOMETRY: ALGORITHMS AND APPLICATIONS

Mark de Berg, Marc van Krveld, Mark Overmars and Otfried Schwarzkopf,

1997

xiii-365

Springer Verlag, Berlin

ISBN 3-540-61270-x

The latest decades have seen a development of the available computing tools. Hence, the interest in the practical solution of geometric problems has been increased as the users has been provided of the needed computational support. This fact have led to the increase in the work on Computational Geometry disciplines as

- *Computer Graphics [CG],*
  - *Computer Aided Design [CAD],*
    - *Computer Aided Manufacturing [CAM],*
      - *Geographic Information System [GIS] and*
        - *Robotics.*

They command the development of new theory, software and hardware. Following the authors, the main aim of the book is to present a modern account, in an unified manner, of these problems. Sixteen chapters are used for doing so.

Chapter s 1, 10-16 deal with the description of techniques for CG in two dimensions and the 13<sup>th</sup> deals with 3-dimension problems. CAD/CAM simulation algorithms are discussed in Chapter 14.

GIS problems are the subject of Chapters 6, 10. It is re-studied in the 16<sup>th</sup>. Robotics is the theme of Chapters 13-15.

The rest of the book [Chapter 1-12] provides the results needed for coping which the whole discipline. It is a good source for courses in Engineering or in Informatics.

The authors provide exercises, notes and comments at the end of each chapter. A list of 345 references is given. They permit to obatin an overview of the whole theory, developed algorithms and applications. 370 illustrations appear throughout the book.

I highly recommend it for specialists dealing with Computational Geometry.

# **RESAMPLING METHODS: A PRACTICAL GUIDE TO DATA ANALYSIS**

Phillip I. Good  
Birkhäuser [1999], xii+269 pp, Boston  
ISBN 0-8176-4091-6  
ISBN 3-7343-4091-6

A naïve reader would perhaps be led to expect to obtain a deep insight in the use of resampling methods. Instead, we find that through the 12 chapters is presented a challenging exposition of the theory of statistics. Resampling procedures are included, successfully, as an alternative for analyzing data in different chapters.

The first chapter presents the methods of Descriptive Statistics and the second one pictures the role of probability as a model for coping with uncertainty. The Binomial distribution is introduced. The third chapter is devoted to Tests of Hypothesis. It discusses the roots of Permutation Tests. Then, the role of the significance level is clearly established. As a result the introduction of Bootstrap procedures is quite natural. The story is that he constructs different tests based on the calculation of the significance level in permutation tests. Hence Bootstrap can provide an estimate of it, if necessary. One and two samples tests are presented. In the following chapter tests of location and scale are discussed when the distribution is Binomial, Poisson or Normal.

What is done in estimation and which are the related risks are the questions that are answered through Chapter five. Again, an original exposition of the problem is developed. Of course, point and interval estimation are discussed but the use of an incorrect model may be another source of error. Then Bootstrap is viewed as a method that permits not only to estimate the error but to establish approximate confidence levels. The logic that sustains the use of Smooth and Bias-corrected Bootstrap is briefly discussed and the corresponding procedures are presented. Chapters 6, 7 and 9 are developed in a more classic way. The eighth chapter is devoted to the design of experiments. The reasoning that suggests to block and to balance is developed with elegance and pedagogical expertise. The role of Bootstrap in unbalanced designs is discussed.

Chapters ten and eleven abord special classes of problems:

*Classification-Discrimination and Survival Reliability methods.*

In the last chapter some of the crucial thoughts, that are posed to the statisticians when a research strategy should be designed, are developed. They go from the decision of using, or not, a parametric approach up to those present if the application of different models presented in the book.

I, personally, expected that the discussion on resampling plans would include a larger analysis on other methods, as Jackknife for example. It is positively biased towards Bootstrap principle. I consider that the introduction of the main ideas of resampling is successfully done. A first or second course on Statistics may utilize this book if the docent plans to introduce these ideas at an early stage. I highly recommends it for coping with this purpose.

Each chapter gives a clue for further readings and a set of exercises. Some macros for the development of programs for computing Bootstrap estimates are listed. They are GAUSS, SAS, S-plus and STATA codes. A list of more than 400 references is provided.

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## **JOHN E. FREUND'S MATHEMATICAL STATISTICS**

**Irwing Miller and Marylees Miller, 1999**

xii + 624

Prentice Hall International Inc., New Jersey  
ISBN 0-13-974155-0

This is an appropriate book for a calculus based two semester course in statistics. It aims to update the previous five editions due to the authorship of J.E. Freund.

A basic knowledge of Calculus is needed for following the exposition but it is not used in proofs but in presenting and deducing concepts, methods and properties.

The first eight chapters permit to organize a course on probability:

*Introduction, Probability, Probability distributions, Probability densities,  
Mathematical expectations, Special probability distributions and densities,  
Functions of random variables, Sampling distributions.*

The other eight chapters deal with statistical theory and methods. A Decision Theory based course may be organized using them:

*Decision Theory, Estimation: Theory, Estimation: applications,  
Hypothesis testing: theory, Hypothesis testing: applications,  
Regression and correlation, Analysis of variance and Non parametric tests.*

If the first chapter is evaded Decision Theory points of view will be of not use but the contents are still easily followed.

The appendixes contain some basic probability theorem and an almost complete set of statistical tables [12]. There is a collection of exercises at the end of each chapter. The responses to the odd-numbered exercises are given.

A more updated list of references was expected but it is basically the last edition's one.

Emphasis has been given to the use of well known statistical packages.

Beyond being a tribute to the pedagogical work of Prof. Freund in Mathematical Statistics the book is self valuable. A modern introductory course may be organized using it as basic text book. For those who have studied and/or actually use the previous Freund's books the present oeuvre permits to update their teaching.

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# ANNALS OF THE INTERNATIONAL SOCIETY OF GAMES 4

M. Bardi, T. Parthasarathy y T.E.S. Raghavan [editores], 1999  
Stochastic and Differential Games. Theory and Numerical Methods.  
ISBN 3-7643-4029-0, 400 páginas.  
Birkhauser, Basel.

El libro consiste de dos partes:

*La primera* es dedicada a juegos diferenciales de suma cero y los métodos numéricos asociados. El primer capítulo fue desarrollado por el difunto Profesor Andrei Subbotin. Se dedica a la discusión de la teoría de los juegos diferenciales posicionales. Se analiza el caso en que existe la solución de la ecuación de Bellman-Isaacs para el juego. Es el caso clásico. Aquel en que la solución es débilmente univariante es tratado al usar la definición de solución minimal en vez de la teoría de viscosidad. Se construyen  $\varepsilon$ -estrategias posicionales que son  $\varepsilon$ -óptimas cuando la función valor no es suave. Son usados quasi-gradientes de la solución de la ecuación de Bellman-Isaac. Este tipo de estrategias son definidas cuando las funciones pueden ser discontinuas.

Soluciones robustas son consideradas y la estabilidad de la retroalimentación es estudiada.

Algunos métodos numéricos son presentados.

P.E.S. Souganidis describe el juego diferencial de suma cero con horizonte finito. Una explicación de las relaciones entre estos juegos y las soluciones viscosas es llevada a cabo. Se analiza la existencia y unicidad de las ecuaciones de primer orden, así como la estabilidad y convergencia de los algoritmos numéricos presentados.

M. Bardi, M. Falconi y P. Soravia abordan el estudio de los juegos de Persecución-Escape. Las soluciones de las funciones valor son interpretadas en el sentido de la viscosidad bajo hipótesis de continuidad. Un algoritmo es propuesto para computar las aproximaciones requeridas. Su convergencia es probada.

P. Cardaliagnet, M. Quincampoix y P. Saint-Pierre estudian problemas de control y juegos diferenciales. Proponen un enfoque para computar la función de valor a partir del Kernel Viable. Un método numérico es propuesto y se establece su convergencia.

*En la segunda parte* se abordan los capítulos sobre Juegos Estocásticos de Suma no cero y sus Aplicaciones.

A. Maitra y W. Sudderth hacen una introducción para hacer aplicaciones de la Teoría de Apuestas en los Juegos Estocásticos. Los problemas que tratan son los de las Apuestas Dejables y no Dejables, así como los Juegos Estocásticos. S.A. Connell, J.A. Filar, W.W. Szczehla y O.J. Vrieze brindan una descripción intuitiva de la aplicación del Análisis de Variedades Complejo a los Juegos de Descuentos Estocásticos. Una semblanza de su desarrollo es llevada a cabo y varios ejemplos ilustrativos son trabajados.

A.S. Nowack y K. Szajowski elaboran un análisis de la situación actual de la teoría de los Juegos Estocásticos de Suma no Cero.

F. Thuysman y O.J. Vrieze por su parte derivan una condición suficiente para la existencia del equilibrio de Juegos Estocásticos Generales. Por último E. Altman modela el enrutamiento en redes usando un Juego Markoviano. Este modela el deseo de obtener una estrategia óptima en el sentido de que esta tenga el mejor desempeño bajo las peores condiciones.

Este libro es de interés particular para los especialistas involucrados con problemas modelables mediante Juegos Dinámicos:

matemáticos aplicados,  
optimizadores e  
ingenieros dedicados al control.

Una visión de la panorámica actual de la temática, así como de esquemas numéricos que permitan la implementación práctica de los métodos discutidos es brindada. Aunque solo el 13 % de los autores no son académicos profesionales hay una marcada orientación hacia la aplicación de los modelos presentados. Esto se evidencia a través de los diversos algoritmos numéricos desarrollados y utilizados en la discusión de ejemplos.

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